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Locality in the Everett Interpretation of Heisenberg-Picture Quantum Mechanics*

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Abstract

Bell's theorem depends crucially on counterfactual reasoning, and is mistakenly interpreted as ruling out a local explanation for the correlations which can be observed between the results of measurements performed on spatially-separated quantum systems. But in fact the Everett interpretation of quantum mechanics, in the Heisenberg picture, provides an alternative local explanation for such correlations. Measurement-type interactions lead, not to many worlds but, rather, to many local copies of experimental systems and the observers who measure their properties. Transformations of the Heisenberg-picture operators corresponding to the properties of these systems and observers, induced by measurement interactions, "label" each copy and provide the mechanism which, e.g., ensures that each copy of one of the observers in an EPRB or GHZM experiment will only interact with the "correct" copy of the other observer(s). The conceptual problem of nonlocality is thus replaced with a conceptual problem of proliferating labels, as correlated systems and observers undergo measurement-type interactions with newly-encountered objects and instruments; it is suggested that this problem may be resolved by considering quantum field theory rather than the quantum mechanics of particles.

1 Introduction

In the paper in which he introduces what has come to be known as the Everett or many-worlds interpretation of quantum mechanics, Everett (1957) states that "fictitious paradoxes like that of Einstein, Podolsky, and Rosen which are concerned with . . . correlated,

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noninteracting systems are easily investigated and clarified in the present scheme." In the Everett interpretation the nonlocal notion of reduction of the wavefunction is eliminated, suggesting that questions of the locality of quantum mechanics might indeed be more easily addressed. On the other hand, while wavefunctions do not suffer reduction in the Everett interpretation, nonlocality nevertheless remains present in many accounts of this formulation. In DeWitt's (1970) often-quoted description, for example, "every quantum transition taking place on every star, in every galaxy, in every remote corner of the universe is splitting our local world on earth into myriads of copies of itself." Contrary to this viewpoint, others argue (Page, 1982; Tipler, 1986, 2000; Albert and Loewer, 1988; Albert, 1992; Vaidman, 1994, 1998, 1999; Price, 1995; Lockwood, 1996; Deutsch, 1996; Deutsch and Hayden, 2000) that the Everett interpretation can in fact *resolve* the apparent contradiction between locality and quantum mechanics. In particular, Deutsch and Hayden (2000) apply the Everett interpretation to quantum mechanics in the Heisenberg picture, and show that in EPRB experiments,[1] information regarding the correlations between systems is encoded in the Heisenberg-picture operators corresponding to the observables of the systems, and is carried from system to system and from place to place in a local manner. The picture which emerges is not one of measurement-type interactions "splitting the universe" but, rather, producing copies of the observers and observed physical systems which have interacted during the (local) measurement process (Tipler, 1986).

The purpose of the present paper is to summarize the formalism of measurement in the Everett interpretation of Heisenberg picture quantum mechanics and its application to the EPRB and GHZM experiments, to emphasize the key aspects of this formulation of quantum mechanics which allow it to circumvent Bell's theorem (Bell, 1964) and to describe the conceptual framework—a "labeled copies interpretation"—which it seems to imply. The information carried in entangled Heisenberg-picture operators governs the nature of correlations observed between the states of entangled systems and the observers who measure them. It is the existence of this mechanism for bringing about, in a local manner, the perfect correlations which are observed, e.g., when the analyzer magnets in the EPRB experiment are parallel, which vitiates the reasoning which otherwise leads one to conclude that either Bell's theorem must hold or nonlocal influences must come into play. Since, in this scenario, even an entity as simple as an electron carries with it for eternity a record of other entities with which it has interacted, this interpretation entails a conceptual difficulty of its own. It is possible that this difficulty may be less severe if quantized fields rather than particles are considered.

In Section 2 below I review the aspects of Bell's theorem most relevant for the EPRB and the GHZM experiments. In Section 3 the Everett model for quantum measurement is reviewed in the original Schrödinger picture formulation as well as in the Heisenberg picture. Section 4 contains an analysis of the EPRB and GHZM experiments from an Everett point of view in the Heisenberg picture. In Section 5 I discuss the manner in which the labeled copies interpretation of quantum mechanics avoids Bell's theorem, the problem of label proliferation, and the possible relevance of quantum field theory for a solution.

[1] Deutsch and Hayden (2000) analyze a variant of the EPRB experiment in which, rather than passing through rotated analyzer magnets, the correlated particles are themselves each independently rotated before their spins are measured. This setup yields the same correlations as the usual one, but allows the flow of information to be tracked explicitly at each step.

2 Bell's Theorem and Counterfactual Reasoning

There are many derivations of the many variants of Bell's theorem; here we review one of the simplest (Farris, 1995). Consider two observers performing Bohm's version (Bohm, 1951) of the Einstein-Podolsky-Rosen (Einstein et al., 1935) experiment (EPRB) on pairs of spin-1/2 particles in the singlet state, using pairs of Stern-Gerlach analyzer magnets which can be independently oriented in one of three directions 120° apart and perpendicular to the particles' line of flight. Define the quantity Q:

$$Q = P_{uu}(0^\circ, 120^\circ) + P_{uu}(120^\circ, 240^\circ) + P_{uu}(240^\circ, 0^\circ), \tag{1}$$

where $P_{uu}(\phi_1, \phi_2)$ is the probability of both observers obtaining the result spin-up from a particle pair when analyzers 1 and 2 are oriented in directions ϕ_1 and ϕ_2 respectively. Each of the three probabilities on the right-hand side of equation (1) can be determined experimentally to any desired degree of accuracy, by performing many repetitions of the EPRB experiment with the analyzers held in the appropriate directions. There is no quantum-mechanical restriction on performing these experiments because, in each case, we are measuring spin components of two *different* particles, so the measurements commute.

However, whenever experiments are performed in which both analyzer magnets have the same orientation ϕ, we observe that

$$P_{uu}(\phi, \phi) = 0 \tag{2}$$

for any and all choices of ϕ. That is, if the analyzer directions are the same, we find that whenever a particle is deflected in one direction by one of the analyzers, its partner is deflected in the opposite direction by the other analyzer. We find that the correlations persist even when we consider only cases in which the analyzer orientations come to be parallel by chance, because they've been chosen at the last possible moment before the particles arrive by some random process (delayed-choice experiment). We are thus compelled to pose a "bothersome question" (Mermin, 1990a): What is the mechanism which brings about these correlations? In answer, we adopt what seems the only explanation open to us: Each particle, even before its spin is measured by the analyzer, carries with it information—"instruction sets," as termed by Mermin (1990a)—determining what its response will be to the analyzer in every possible orientation.

Having accepted this explanation, our doom is sealed. For if this explanation holds, it is a well-defined notion to talk about what *would* have happened if an analyzer had been oriented other than as it actually was in any given experiment. That is, we define the quantity $P^{(1)}_{IS-ud}(\phi_1, \phi_2)$ to be the probability that particle 1 is carrying instructions to be deflected up by an analyzer with orientation ϕ_1, and at the same time is carrying instructions to be deflected down by an analyzer at orientation ϕ_2. This cannot be measured directly; but, by the reasoning above, it has the value

$$P^{(1)}_{IS-ud}(\phi_1, \phi_2) = P_{uu}(\phi_1, \phi_2). \tag{3}$$

So, using this in (1),

$$Q = P^{(1)}_{IS-ud}(0^\circ, 120^\circ) + P^{(1)}_{IS-ud}(120^\circ, 240^\circ) + P^{(1)}_{IS-ud}(240^\circ, 0^\circ), \tag{4}$$

and since the probabilities which are being added on the right-hand side of (4) are of mutually exclusive events (e.g., particle 1 is carrying instructions to be deflected *either* up *or* down by a magnet with orientation 120°) we conclude

$$Q \leq 1. \tag{5}$$

This inequality contradicts the prediction obtained from a quantum-mechanical calculation of Q (see, e.g., Section 4.1.2 below), and it is the latter which is borne out by actual experiments (Aspect et al., 1982; Weihs et al., 1998).

The arguments leading to Mermin's (1990b,c) three-particle version of the Greenberger-Horne-Zeilinger (Greenberger et al., 1989,1990) experiment (GHZM) are similarly based on the need to explain perfect correlations. In this case, the fact that the results of spin measurements made on one particle correlate perfectly with those made on two other particles drives us to conclude that the results of spin measurements on all three particles are governed by instruction sets. The three particles in question each have spin-1/2 and travel outward from a common source in three coplanar directions. The spin of each particle is measured by an analyzer magnet that can be oriented at any angle in the plane perpendicular to the corresponding particle's line of flight. (The 0° direction is perpendicular to the common plane of the particles' motion.) Quantum mechanics predicts (see Section 4.2.2) that

$$P_{eu}(0^\circ, 90^\circ, 90^\circ) = P_{eu}(90^\circ, 0^\circ, 90^\circ) = P_{eu}(90^\circ, 90^\circ, 0^\circ) = 0, \tag{6}$$

where $P_{eu}(\phi_1, \phi_2, \phi_3,)$ is the probability that an even number of spin measurements will be up. Enumeration of the possible instruction sets that could account for these results (Mermin, 1990b) leads to the conclusion—here, not an inequality, but an equality—that

$$P_{eu}(0^\circ, 0^\circ, 0^\circ) = 0 \quad \text{(instruction set prediction).} \tag{7}$$

However, a direct quantum-mechanical calculation of this quantity gives a probability , not of zero, but of unity (see Section 4.2.2).

3 Everett's Measurement Model

3.1 Schrödinger Picture

Everett's paper presents a model of ideal measurements in quantum mechanics. Consider a physical system $\mathcal{S}$ and an observer[2] $\mathcal{O}$. The space of states of $\mathcal{S}$ is spanned by the eigenstates of a Hermitian operator $\widehat{a}$ with eigenvalues α_i:

$$\widehat{a}|\mathcal{S};\alpha_i\rangle = \alpha_i|\mathcal{S};\alpha_i\rangle, \quad i = 1,\ldots,N. \tag{8}$$

For simplicity we will assume that the eigenvalues α_i are nondegenerate. Here and below, unless indicated otherwise, all operators are time-independent Schrödinger-picture operators.

[2] The observer, of course, is also a physical system!

The space of states of $\mathcal{O}$ is spanned by the eigenstates of a Hermitian operator $\widehat{b}$ with eigenvalues β_I:

$$\widehat{b}|\mathcal{O};\beta_I\rangle = \beta_I|\mathcal{O};\beta_I\rangle, \quad I = 0,\ldots,N. \tag{9}$$

The eigenvalues β_I correspond to distinct "states of belief" of the observer $\mathcal{O}$ concerning the results of measurements made on the system $\mathcal{S}$ ($\mathcal{O}$ could of course be a computer or other machine rather than a conscious human), so we can take them to be nondegenerate, with β_0 corresponding to the state of ignorance (no measurement yet made).

The interaction corresponding to the measurement of $\mathcal{S}$ by $\mathcal{O}$ is represented by a unitary time-evolution operator $\widehat{U}_M$ acting in the product space of the state spaces of $\mathcal{S}$ and $\mathcal{O}$. In order to correspond to an ideal measurement (the only type considered in this paper), $\widehat{U}_M$ must have the property that if at time t_1 $\mathcal{O}$ is in a state of ignorance and $\mathcal{S}$ is in a state where the quantity represented by $\widehat{a}$ definitely has the value α_i—i.e.,

$$|\psi;t_1\rangle = |\mathcal{O};\beta_0\rangle|\mathcal{S};\alpha_i\rangle, \tag{10}$$

so

$$\begin{aligned}\widehat{A}|\psi;t_1\rangle &= \alpha_i|\psi;t_1\rangle, &(11)\\ \widehat{B}|\psi;t_1\rangle &= \beta_0|\psi;t_1\rangle, &(12)\end{aligned}$$

where

$$\begin{aligned}\widehat{A} &\equiv \widehat{a}\otimes\widehat{I}_{\mathcal{S}}, &(13)\\ \widehat{B} &\equiv \widehat{I}_{\mathcal{O}}\otimes\widehat{b}, &(14)\\ \widehat{I}_{\mathcal{S}} &\equiv \text{identity operator in space of states of } \mathcal{S}, &(15)\\ \widehat{I}_{\mathcal{O}} &\equiv \text{identity operator in space of states of } \mathcal{O} &(16)\end{aligned}$$

—then the action of $\widehat{U}_M$ is given by

$$|\psi;t_2\rangle = \widehat{U}|\psi;t_1\rangle = |\mathcal{O};\beta_i\rangle|\mathcal{S};\alpha_i\rangle. \tag{17}$$

Since $\widehat{U}_M$ is a linear operator, its effect on a state in which $\mathcal{O}$ is ignorant and $\mathcal{S}$ is in an arbitrary superposition of $\widehat{a}$ eigenstates,

$$|\psi;t_1\rangle = |\mathcal{O};\beta_0\rangle(\sum_i c_i|\mathcal{S};\alpha_i\rangle, \tag{18}$$

is

$$|\psi;t_2\rangle = \widehat{U}_M|\psi;t_1\rangle = \sum_i c_i|\mathcal{O};\beta_i\rangle|\mathcal{S};\alpha_i\rangle. \tag{19}$$

The state $|\psi;t_2\rangle$ is said to be "entangled," since it is not a product of states in the respective state spaces of $\mathcal{S}$ and $\mathcal{O}$.

Therefore

$$\widehat{U}_M = \sum_i \widehat{u}_i\otimes\widehat{\Pi}_i, \tag{20}$$

where $\widehat{\Pi}_i$ is the projection operator into the i^{th} $\widehat{a}$ eigenstate of $\mathcal{S}$,

$$\widehat{\Pi}_i \equiv |\mathcal{S};\alpha_i\rangle\langle\mathcal{S};\alpha_i|, \tag{21}$$

and $\widehat{u}_i$ are unitary operators in the space of states of $\mathcal{O}$ with the property

$$\widehat{u}_i|\mathcal{O};\beta_0\rangle = |\mathcal{O};\beta_i\rangle, \quad i = 1,\ldots,N. \tag{22}$$

The action of $\widehat{u}_i$ on states $|\mathcal{O};\beta_I\rangle$, $I \neq 0$, will not play a role in what follows. (In Section 4.1 below we give a specific example of operators $\widehat{u}_i$ for the case $N = 2$.)

3.2 Heisenberg Picture

In the Heisenberg picture, time dependence is carried by the operators. Heisenberg-picture operators will be distinguished by explicit time arguments. At the initial time t_0 the Heisenberg-picture operators are identical to their Schrödinger-picture counterparts:

$$\begin{array}{rcccl} \widehat{A}(t_0) & = & \widehat{A} & = & \widehat{I}_{\mathcal{O}} \otimes \widehat{a}, \\ \widehat{B}(t_0) & = & \widehat{B} & = & \widehat{b} \otimes \widehat{I}_{\mathcal{S}}. \end{array} \tag{23}$$

At time t_2, after $\mathcal{O}$ has measured $\mathcal{S}$, these operators become, respectively,

$$\begin{array}{rcl} \widehat{A}(t_2) & = & \widehat{U}_M^\dagger\, \widehat{A}\, \widehat{U}_M, \\ \widehat{B}(t_2) & = & \widehat{U}_M^\dagger\, \widehat{B}\, \widehat{U}_M. \end{array} \tag{24}$$

Here $t_2 > t_1 > t_0$, and, for now, it is assumed that no interaction takes place between t_0 and t_1.

From (13-16), (20), (23), and (24),

$$\widehat{A}(t_2) = \widehat{I}_{\mathcal{O}} \otimes \widehat{a}, \tag{25}$$

$$\widehat{B}(t_2) = \sum_i \widehat{u}_i^\dagger \widehat{b} \widehat{u}_i \otimes \widehat{\Pi}_i, \tag{26}$$

since

$$\widehat{u}_i^\dagger \widehat{u}_i = \widehat{I}_{\mathcal{O}}, \tag{27}$$

$$\widehat{\Pi}_i^\dagger = \widehat{\Pi}_i, \tag{28}$$

$$\widehat{\Pi}_i \widehat{\Pi}_j = \widehat{\Pi}_i \delta_{ij}, \tag{29}$$

$$\widehat{a} = \sum_i \alpha_i \widehat{\Pi}_i. \tag{30}$$

In the Heisenberg picture operator the $\mathcal{S}$ observable $\widehat{A}(t)$ is the same after the measurement as before. (This will not be the case in general; see Section 4.1.1.) However, the $\mathcal{O}$ observable $\widehat{B}(t)$ has become entangled with $\mathcal{S}$, in that it is no longer in the form (14), the tensor product of an operator acting on $\mathcal{O}$ states with the identity operator on $\mathcal{S}$ states, but instead acts nontrivially on the states of $\mathcal{S}$. This is the hallmark of entanglement in the Heisenberg picture (d'Espagnat, 1995, Section 10.8). The Heisenberg picture state vector, on the other hand, remains at all times equal to the nonentangled Schrödinger picture time-t_0 state vector (18).

4 EPRB and GHZM Experiments

In EPRB and GHZM experiments, the particles are prepared in a state in which they are entangled with each other before measurement. A spin component of each of the particles is subsequently measured by a corresponding analyzer magnet which can be at one of several orientations. As in the previous section, the action of the unitary time evolution operator will first be determined by working in the Schrödinger picture and subsequently used to compute the form of the time-dependent operators in the Heisenberg picture. For purposes of computational convenience, all operator eigenstates employed will be time-independent eigenstates of time-independent operators—i.e., Schrödinger-picture eigenstates.

4.1 EPRB Experiment

The two particles are denoted $\mathcal{S}^{(p)}$ and the two observers $\mathcal{O}^{(p)}$, $p = 1, 2$. The space of states of $\mathcal{S}^{(p)}$ is spanned by eigenstates of the Hermitian operator $\widehat{a}^{(p)}$. In this case $\widehat{a}^{(p)}$ is the z component of the p^{th}-particle spin operator

$$\widehat{a}^{(p)} = \widehat{\sigma}_z^{(p)}, \quad p = 1, 2, \tag{31}$$

where spin is measured in units of $\hbar/2$. The $\mathcal{S}^{(p)}$ eigenbasis thus given by

$$\widehat{a}^{(p)}|\mathcal{S}^{(p)}; \alpha_i\rangle = \alpha_i|\mathcal{S}^{(p)}; \alpha_i\rangle, \quad i, p = 1, 2, \tag{32}$$

where

$$\alpha_1 = +1, \qquad \alpha_2 = -1. \tag{33}$$

The space of states of $\mathcal{O}^{(p)}$ is spanned by

$$\widehat{b}^{(p)}|\mathcal{O}^{(p)}; \beta_I\rangle = \beta_I|\mathcal{O}^{(p)}; \beta_I\rangle, \quad I = 0, 1, 2. \tag{34}$$

The eigenvalue β_0 corresponds to the ignorant state of the observer. Eigenvalues β_1 and β_2 correspond to the observer $\mathcal{O}^{(p)}$ having measured the spin of particle $\mathcal{S}^{(p)}$ to be respectively up or down.

Since the EPRB experiment involves measurements in several directions, we consider measurement interactions by the observers $\mathcal{O}^{(p)}$ using analyzer magnets oriented along arbitrary independent directions denoted by unit vectors $\vec{n}^{(p)}$,

$$\vec{n}^{(p)} = (n_x^{(p)}, n_y^{(p)}, n_z^{(p)}) = (\sin\theta^{(p)}\cos\phi^{(p)}, \sin\theta^{(p)}\sin\phi^{(p)}, \cos\theta^{(p)}). \tag{35}$$

The time-evolution operators corresponding to these measurements are therefore

$$\begin{array}{rcl} \widehat{U}^{(1)}_{M,\vec{n}^{(1)}} & = & \sum_i \widehat{u}_i^{(1)} \otimes \widehat{I}_{\mathcal{O}}^{(2)} \otimes \widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{I}_{\mathcal{S}}^{(2)}, \\ \widehat{U}^{(2)}_{M,\vec{n}^{(2)}} & = & \sum_i \widehat{I}_{\mathcal{O}}^{(1)} \otimes \widehat{u}_i^{(2)} \otimes \widehat{I}_{\mathcal{S}}^{(1)} \otimes \widehat{\Pi}^{(2)}_{i,\vec{n}^{(2)}}, \end{array} \tag{36}$$

where $\widehat{\Pi}^{(p)}_{i,\vec{n}^{(p)}}$ is the projection operator into the i^{th} eigenstate of particle p along direction $\vec{n}^{(p)}$,

$$\widehat{\Pi}^{(p)}_{i,\vec{n}^{(p)}} = |\mathcal{S}^{(p)}, \vec{n}^{(p)}; \alpha_i\rangle\langle\mathcal{S}^{(p)}, \vec{n}^{(p)}; \alpha_i|. \tag{37}$$

In terms of spin eigenstates defined with respect to the z axis (Greenberger et al., 1990, Appendix A),

$$
\begin{aligned}
|\mathcal{S}^{(p)},\vec{n}^{(p)};\alpha_1\rangle &= \exp(-i\phi/2)\cos(\theta/2)|\mathcal{S}^{(p)};\alpha_1\rangle + \exp(i\phi/2)\sin(\theta/2)|\mathcal{S}^{(p)};\alpha_2\rangle, \\
|\mathcal{S}^{(p)},\vec{n}^{(p)};\alpha_2\rangle &= -\exp(-i\phi/2)\sin(\theta/2)|\mathcal{S}^{(p)};\alpha_1\rangle + \exp(i\phi/2)\cos(\theta/2)|\mathcal{S}^{(p)};\alpha_2\rangle,
\end{aligned}
\tag{38}
$$

so

$$
\begin{aligned}
\widehat{\Pi}^{(p)}_{1,\vec{n}^{(p)}} &= \cos^2(\theta^{(p)}/2)\,\widehat{\Pi}^{(p)}_1 + \sin^2(\theta^{(p)}/2)\,\widehat{\Pi}^{(p)}_2 \\
&\quad + \sin\theta^{(p)}\left(\exp(-i\phi^{(p)})\,\widehat{T}^{(p)}_{1-2} + \exp(i\phi^{(p)})\,\widehat{T}^{(p)}_{2-1}\right)/2, \\
\widehat{\Pi}^{(p)}_{2,\vec{n}^{(p)}} &= \sin^2(\theta^{(p)}/2)\,\widehat{\Pi}^{(p)}_1 + \cos^2(\theta^{(p)}2)\,\widehat{\Pi}^{(p)}_2 \\
&\quad - \sin\theta^{(p)}\left(\exp(-i\phi^{(p)})\,\widehat{T}^{(p)}_{1-2} + \exp(i\phi^{(p)})\,\widehat{T}^{(p)}_{2-1}\right)/2,
\end{aligned}
\tag{39}
$$

where

$$
\begin{aligned}
\widehat{\Pi}^{(p)}_1 &= |\mathcal{S}^{(p)};\alpha_1\rangle\langle\mathcal{S}^{(p)};\alpha_1|, \qquad & \widehat{\Pi}^{(p)}_2 &= |\mathcal{S}^{(p)};\alpha_2\rangle\langle\mathcal{S}^{(p)};\alpha_2|, \\
\widehat{T}^{(p)}_{1-2} &= |\mathcal{S}^{(p)};\alpha_1\rangle\langle\mathcal{S}^{(p)};\alpha_2|, \qquad & \widehat{T}^{(p)}_{2-1} &= |\mathcal{S}^{(p)};\alpha_2\rangle\langle\mathcal{S}^{(p)};\alpha_1|.
\end{aligned}
\tag{40}
$$

The operators $\hat{u}^{(p)}_i$ have the properties

$$
\begin{aligned}
\hat{u}^{(p)}_1|\mathcal{O}^{(p)};\beta_I\rangle &= |\mathcal{O}^{(p)};\beta_{I+1\,\mathrm{mod}\,3}\rangle, \\
\hat{u}^{(p)}_2|\mathcal{O}^{(p)};\beta_I\rangle &= |\mathcal{O}^{(p)};\beta_{I-1\,\mathrm{mod}\,3}\rangle.
\end{aligned}
\tag{41}
$$

Of these properties, the relevant ones for what follows are those for $I = 0$:

$$
\begin{aligned}
\hat{u}^{(p)}_1|\mathcal{O}^{(p)};\beta_0\rangle &= |\mathcal{O}^{(p)};\beta_1\rangle, \\
\hat{u}^{(p)}_2|\mathcal{O}^{(p)};\beta_0\rangle &= |\mathcal{O}^{(p)};\beta_2\rangle.
\end{aligned}
\tag{42}
$$

In the Heisenberg picture, the average of the product of the results of the measurements made by $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$ at time t_2 is

$$
\langle B^{(1)}B^{(2)}\rangle(t_2) = \langle\psi,t_0|\widehat{B}^{(1)}(t_2)\widehat{B}^{(2)}(t_2)|\psi,t_0\rangle, \tag{43}
$$

where $|\psi,t_0\rangle$ is the state of $\mathcal{O}^{(1)}$,$\mathcal{O}^{(2)}$, $\mathcal{S}^{(1)}$ and $\mathcal{S}^{(2)}$ at the initial time t_0, and the $\widehat{B}^{(p)}(t_2)$ are the operators on states of $\mathcal{O}^{(p)}$ after both measurements have been made. (From (36) we see that $\widehat{U}^{(1)}_{M,\vec{n}^{(1)}}$ and $\widehat{U}^{(2)}_{M,\vec{n}^{(2)}}$ commute, so the order in which the measurements are made is immaterial.) At time t_0,

$$
\begin{aligned}
\widehat{A}^{(1)} &= \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{a}^{(1)} \otimes \widehat{I}^{(2)}_{\mathcal{S}}, \\
\widehat{A}^{(2)} &= \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{a}^{(2)}, \\
\widehat{B}^{(1)} &= \widehat{b}^{(1)} \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{I}^{(2)}_{\mathcal{S}}, \\
\widehat{B}^{(2)} &= \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{b}^{(2)} \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{I}^{(2)}_{\mathcal{S}}.
\end{aligned}
\tag{44}
$$

For the initial-time Heisenberg picture state, we will use the state in which both observers are ignorant and each of the pair of particles has a well-defined spin, with particle 1 up with respect to the z axis and particle 2 down:

$$
|\psi,t_0\rangle = |\mathcal{O}^{(1)};\beta_0\rangle|\mathcal{O}^{(2)};\beta_0\rangle|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle. \tag{45}
$$

4.1.1 Nonentangled Particles

First consider the case in which the only interactions which take place subsequent to time t_0 are the measurements of the two particles by the two observers. The time evolution operator from time t_0 to time t_2 is then

$$\widehat{U} \equiv \widehat{U}^{(2)}_{M,\vec{n}^{(2)}} \widehat{U}^{(1)}_{M,\vec{n}^{(1)}}. \tag{46}$$

The Heisenberg-picture observables at time t_2,

$$\begin{array}{rclcll} \widehat{A}^{(p)}(t_2) & = & \widehat{U}^\dagger \widehat{A}^{(p)} \widehat{U} & = & \left(\widehat{U}^{(1)\dagger}_{M,\vec{n}^{(1)}} \widehat{U}^{(2)\dagger}_{M,\vec{n}^{(2)}}\right) \widehat{A}^{(p)} \left(\widehat{U}^{(2)}_{M,\vec{n}^{(2)}} \widehat{U}^{(1)}_{M,\vec{n}^{(1)}}\right), & p = 1,2, \\ \widehat{B}^{(p)}(t_2) & = & \widehat{U}^\dagger \widehat{B}^{(p)} \widehat{U} & = & \left(\widehat{U}^{(1)\dagger}_{M,\vec{n}^{(1)}} \widehat{U}^{(2)\dagger}_{M,\vec{n}^{(2)}}\right) \widehat{B}^{(p)} \left(\widehat{U}^{(2)}_{M,\vec{n}^{(2)}} \widehat{U}^{(1)}_{M,\vec{n}^{(1)}}\right), & p = 1,2, \end{array} \tag{47}$$

are therefore, using (36),

$$\widehat{A}^{(1)}(t_2) = \sum_{i,j} \widehat{u}^{(1)\dagger}_i \widehat{u}^{(1)}_j \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \widehat{a}^{(1)} \widehat{\Pi}^{(1)}_{j,\vec{n}^{(1)}} \otimes \widehat{I}^{(2)}_{\mathcal{S}}, \tag{48}$$

$$\widehat{A}^{(2)}(t_2) = \sum_{i,j} \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{u}^{(2)\dagger}_i \widehat{u}^{(2)}_j \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{\Pi}^{(2)}_{i,\vec{n}^{(2)}} \widehat{a}^{(2)} \widehat{\Pi}^{(2)}_{j,\vec{n}^{(2)}}, \tag{49}$$

$$\widehat{B}^{(1)}(t_2) = \sum_{i} \widehat{u}^{(1)\dagger}_i \widehat{b}^{(1)} \widehat{u}^{(1)}_i \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{I}^{(2)}_{\mathcal{S}}, \tag{50}$$

$$\widehat{B}^{(2)}(t_2) = \sum_{i} \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{u}^{(2)\dagger}_i \widehat{b}^{(2)} \widehat{u}^{(2)}_i \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{\Pi}^{(2)}_{i,\vec{n}^{(2)}}. \tag{51}$$

We see that in this case $\mathcal{S}^{(p)}$ as well as $\mathcal{O}^{(p)}$ are entangled. From (32)-(34), (39), (40), (42), (43), (45), (50) and (51), the average value of the product of the spin measurements is

$$\begin{array}{l} \langle B^{(1)} B^{(2)} \rangle(t_2) \\ = \langle \psi, t_0 | \sum_{i,j} \widehat{u}^{(1)\dagger}_i \widehat{b}^{(1)} \widehat{u}^{(1)}_i \otimes \widehat{u}^{(2)\dagger}_j \widehat{b}^{(2)} \widehat{u}^{(2)}_j \otimes \widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{j,\vec{n}^{(2)}} | \psi, t_0 \rangle \\ = \sum_{i,j} \beta_i \beta_j \langle \mathcal{S}^{(1)}; \alpha_1 | \langle \mathcal{S}^{(2)}; \alpha_2 | \widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{j,\vec{n}^{(2)}} | \mathcal{S}^{(1)}; \alpha_1 \rangle | \mathcal{S}^{(2)}; \alpha_2 \rangle \\ = \left(\beta_1 \cos^2(\theta^{(1)}/2) + \beta_2 \sin^2(\theta^{(1)}/2)\right) \left(\beta_1 \sin^2(\theta^{(2)}/2) + \beta_2 \cos^2(\theta^{(2)}/2)\right), \end{array} \tag{52}$$

while the individual expected spin measurements are

$$\begin{array}{rcl} \langle B^{(1)} \rangle(t_2) & = & \langle \psi, t_0 | \widehat{B}^{(1)}(t_2) | \psi, t_0 \rangle \\ & = & \langle \psi, t_0 | \sum_i \widehat{u}^{(1)\dagger}_i \widehat{b}^{(1)} \widehat{u}^{(1)}_i \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{I}^{(2)}_{\mathcal{S}} | \psi, t_0 \rangle \\ & = & \beta_1 \cos^2(\theta^{(1)}/2) + \beta_2 \sin^2(\theta^{(1)}/2), \\ \langle B^{(2)} \rangle(t_2) & = & \langle \psi, t_0 | \widehat{B}^{(2)}(t_2) | \psi, t_0 \rangle \\ & = & \langle \psi, t_0 | \sum_i \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{u}^{(2)\dagger}_i \widehat{b}^{(2)} \widehat{u}^{(2)}_i \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{\Pi}^{(2)}_{i,\vec{n}^{(2)}} | \psi, t_0 \rangle \\ & = & \beta_1 \sin^2(\theta^{(2)}/2) + \beta_2 \cos^2(\theta^{(2)}/2). \end{array} \tag{53}$$

So,

$$\langle B^{(1)} B^{(2)} \rangle(t_2) = \langle B^{(1)} \rangle(t_2) \langle B^{(2)} \rangle(t_2), \tag{54}$$

as of course should be the case for measurements of independent systems.

4.1.2 Entangled Particles

Now suppose that between times t_0 and t_1 an interaction $\widehat{U}_E$ occurs between the two particles which takes the state $|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle$ to the singlet state. Specifically, let

$$\widehat{U}_E = \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{u}_E, \tag{55}$$

where

$$\begin{aligned}
\widehat{u}_E|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_1\rangle &= |\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_1\rangle, &(56)\\
\widehat{u}_E|\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_1\rangle &= \left(|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle + |\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_1\rangle\right)/\sqrt{2}, &(57)\\
\widehat{u}_E|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle &= \left(|\mathcal{S}^{(1)};\alpha_1\rangle|\mathcal{S}^{(2)};\alpha_2\rangle - |\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_1\rangle\right)/\sqrt{2}, &(58)\\
\widehat{u}_E|\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_2\rangle &= |\mathcal{S}^{(1)};\alpha_2\rangle|\mathcal{S}^{(2)};\alpha_2\rangle. &(59)
\end{aligned}$$

The time evolution operator from time t_0 to time t_2 is in this case

$$\widehat{U}' \equiv \widehat{U}^{(2)}_{M,\vec{n}^{(2)}}\widehat{U}^{(1)}_{M,\vec{n}^{(1)}}\widehat{U}_E, \tag{60}$$

so the Heisenberg-picture observables at time t_2,

$$\begin{aligned}
\widehat{A}^{(p)\prime}(t_2) &= \widehat{U}'^{\dagger}\widehat{A}^{(p)}\widehat{U}' = \left(\widehat{U}^{\dagger}_E\widehat{U}^{(1)\dagger}_{M,\vec{n}^{(1)}}\widehat{U}^{(2)\dagger}_{M,\vec{n}^{(2)}}\right)\widehat{A}^{(p)}\left(\widehat{U}^{(2)}_{M,\vec{n}^{(2)}}\widehat{U}^{(1)}_{M,\vec{n}^{(1)}}\widehat{U}_E\right), \quad p=1,2,\\
\widehat{B}^{(p)\prime}(t_2) &= \widehat{U}'^{\dagger}\widehat{B}^{(p)}\widehat{U}' = \left(\widehat{U}^{\dagger}_E\widehat{U}^{(1)\dagger}_{M,\vec{n}^{(1)}}\widehat{U}^{(2)\dagger}_{M,\vec{n}^{(2)}}\right)\widehat{B}^{(p)}\left(\widehat{U}^{(2)}_{M,\vec{n}^{(2)}}\widehat{U}^{(1)}_{M,\vec{n}^{(1)}}\widehat{U}_E\right), \quad p=1,2,
\end{aligned} \tag{61}$$

become

$$\begin{aligned}
\widehat{A}^{(1)\prime}(t_2) &= \sum_{i,j} \widehat{u}^{(1)\dagger}_i\widehat{u}^{(1)}_j \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{u}^{\dagger}_E\left(\widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}}\widehat{a}^{(1)}\widehat{\Pi}^{(1)}_{j,\vec{n}^{(1)}} \otimes \widehat{I}^{(2)}_{\mathcal{S}}\right)\widehat{u}_E, &(62)\\
\widehat{A}^{(2)\prime}(t_2) &= \sum_{i,j} \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{u}^{(2)\dagger}_i\widehat{u}^{(2)}_j \otimes \widehat{u}^{\dagger}_E\left(\widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{\Pi}^{(2)}_{i,\vec{n}^{(2)}}\widehat{a}^{(2)}\widehat{\Pi}^{(2)}_{j,\vec{n}^{(2)}}\right)\widehat{u}_E, &(63)\\
\widehat{B}^{(1)\prime}(t_2) &= \sum_{i} \widehat{u}^{(1)\dagger}_i\widehat{b}^{(1)}\widehat{u}^{(1)}_i \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{u}^{\dagger}_E\left(\widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{I}^{(2)}_{\mathcal{S}}\right)\widehat{u}_E, &(64)\\
\widehat{B}^{(2)\prime}(t_2) &= \sum_{i} \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{u}^{(2)\dagger}_i\widehat{b}^{(2)}\widehat{u}^{(2)}_i \otimes \widehat{u}^{\dagger}_E\left(\widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{\Pi}^{(2)}_{i,\vec{n}^{(2)}}\right)\widehat{u}_E. &(65)
\end{aligned}$$

Using (32)-(34), (39)-(40), (42), (43), (45), (64) and (65),

$$\begin{aligned}
&\langle B^{(1)\prime}B^{(2)\prime}\rangle(t_2)\\
&= \langle\psi,t_0|\ \textstyle\sum_{i,j}\widehat{u}^{(1)\dagger}_i\widehat{b}^{(1)}\widehat{u}^{(1)}_i \otimes \widehat{u}^{(2)\dagger}_j\widehat{b}^{(2)}\widehat{u}^{(2)}_j \otimes \widehat{u}^{\dagger}_E\left(\widehat{\Pi}^{(1)}_{i,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{j,\vec{n}^{(2)}}\right)\widehat{u}_E|\psi,t_0\rangle\\
&= \left((\beta_1+\beta_2)^2 - (\beta_1-\beta_2)^2\,\vec{n}^{(1)}\cdot\vec{n}^{(2)}\right)/4.
\end{aligned} \tag{66}$$

If the eigenvalues labeling the observers' states of awareness are chosen to be $\beta_i = \alpha_i$, (66) takes the well-known form (Greenberger et al., 1990, Appendix B)

$$\langle B^{(1)\prime}B^{(2)\prime}\rangle(t_2)_{\beta_1=1,\,\beta_2=-1} = -\vec{n}^{(1)}\cdot\vec{n}^{(2)}. \tag{67}$$

If $\beta_1 = 1$ and $\beta_2 = 0$, eq.(66) is equal to the probability that both observers find the particles they measure to be deflected in the spin-up direction:

$$\begin{aligned} \langle B^{(1)\prime} B^{(2)\prime}\rangle(t_2)_{\beta_1=1,\,\beta_2=0} &= P_{uu}(\vec{n}^{(1)},\vec{n}^{(2)}) \\ &= \left(1-\vec{n}^{(1)}\cdot\vec{n}^{(2)}\right)/4. \end{aligned} \tag{68}$$

If the angle between $\vec{n}^{(1)}$ and $\vec{n}^{(2)}$is 120°, (68) has the value 3/8, so the quantum-mechanical prediction for Q in eq. (1) is

$$Q = 9/8, \tag{69}$$

contradicting the prediction (5) that $Q \leq 1$.

4.2 GHZM Experiment

We now consider three particles $\mathcal{S}^{(p)}$ and their corresponding observers $\mathcal{O}^{(p)}$, $p = 1, 2, 3$. In addition, we will explicitly introduce an additional observer $\mathcal{O}^{(0)}$ who ascertains the states of awareness of the three observers $\mathcal{O}^{(1)}$, $\mathcal{O}^{(2)}$ and $\mathcal{O}^{(3)}$, after they have performed their respective spin measurements. The space of states of $\mathcal{O}^{(0)}$ is spanned by the eigenstates of the Hermitian operator $\widehat{G}$:

$$\widehat{G} = \widehat{g} \otimes \widehat{I}^{(1)}_{\mathcal{O}} \otimes \widehat{I}^{(2)}_{\mathcal{O}} \otimes \widehat{I}^{(3)}_{\mathcal{O}} \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{I}^{(2)}_{\mathcal{S}} \otimes \widehat{I}^{(3)}_{\mathcal{S}}, \tag{70}$$

where

$$\widehat{g}|\mathcal{O}^{(0)};\gamma_I\rangle = \gamma_I|\mathcal{O}^{(0)};\gamma_I\rangle, \quad I = 0, 1, 2. \tag{71}$$

We want γ_0 to correspond to ignorance, while γ_1 and γ_2 correspond respectively to $\mathcal{O}^{(0)}$ having determined that $\mathcal{O}^{(p)}$, $p = 1, 2, 3$ have observed an odd or even number of spin-up results. The interaction corresponding to the measurement of $\mathcal{O}^{(p)}$ by $\mathcal{O}^{(0)}$ is therefore

$$\widehat{V} = \sum_i \widehat{v}_i \otimes \widehat{P}_i \otimes \widehat{I}^{(1)}_{\mathcal{S}} \otimes \widehat{I}^{(2)}_{\mathcal{S}} \otimes \widehat{I}^{(3)}_{\mathcal{S}}, \tag{72}$$

where

$$\widehat{v}_i|\mathcal{O}^{(0)};\gamma_0\rangle = |\mathcal{O}^{(0)};\gamma_i\rangle, \quad i = 1, 2. \tag{73}$$

and $\widehat{P}_i$, $i = 1, 2$ are the projection operators into the spaces of states in which the three observers $\mathcal{O}^{(p)}$ perceive, respectively, an odd and an even number of spin-up results:

$$\begin{aligned} \widehat{P}_1 &= \widehat{p}^{(1)}_1 \otimes \widehat{p}^{(2)}_2 \otimes \widehat{p}^{(3)}_2 + \widehat{p}^{(1)}_2 \otimes \widehat{p}^{(2)}_1 \otimes \widehat{p}^{(3)}_2 + \widehat{p}^{(1)}_2 \otimes \widehat{p}^{(2)}_2 \otimes \widehat{p}^{(3)}_1 + \\ &\quad \widehat{p}^{(1)}_1 \otimes \widehat{p}^{(2)}_1 \otimes \widehat{p}^{(3)}_1, \end{aligned} \tag{74}$$

$$\begin{aligned} \widehat{P}_2 &= \widehat{p}^{(1)}_1 \otimes \widehat{p}^{(2)}_1 \otimes \widehat{p}^{(3)}_2 + \widehat{p}^{(1)}_1 \otimes \widehat{p}^{(2)}_2 \otimes \widehat{p}^{(3)}_1 + \widehat{p}^{(1)}_2 \otimes \widehat{p}^{(2)}_1 \otimes \widehat{p}^{(3)}_1 + \\ &\quad \widehat{p}^{(1)}_2 \otimes \widehat{p}^{(2)}_2 \otimes \widehat{p}^{(3)}_2, \end{aligned} \tag{75}$$

where

$$\widehat{p}^{(p)}_i = |\mathcal{O}^{(p)};\beta_i\rangle\langle\mathcal{O}^{(p)};\beta_i|, \quad p = 1, 2, 3, \quad i = 1, 2. \tag{76}$$

If the measurement by $\mathcal{O}^{(0)}$ of $\mathcal{O}^{(p)}$ takes place between times t_2 and $t_3 > t_2$,

$$\widehat{G}(t_3) = \left(\widehat{U}_E^\dagger\, \widehat{U}_M^{(1)\dagger}\, \widehat{U}_M^{(2)\dagger}\, \widehat{U}_M^{(3)\dagger}\, \widehat{V}^\dagger\right) \widehat{G} \left(\widehat{V}\, \widehat{U}_M^{(3)}\, \widehat{U}_M^{(2)}\, \widehat{U}_M^{(1)}\, \widehat{U}_E\right) \tag{77}$$

$$= \sum_{i,j,k,l} \widehat{v}_i^\dagger \widehat{g} \widehat{v}_i \otimes \widehat{u}_j^{(1)\dagger} \widehat{u}_k^{(2)\dagger} \widehat{u}_l^{(3)\dagger} \widehat{P}_i\, \widehat{u}_l^{(3)} \widehat{u}_k^{(2)} \widehat{u}_j^{(1)} \otimes \widehat{u}_E^\dagger \left(\widehat{\Pi}^{(1)}_{j,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{k,\vec{n}^{(2)}} \otimes \widehat{\Pi}^{(3)}_{l,\vec{n}^{(3)}}\right) \widehat{u}_E \tag{78}$$

where, as in the previous section, $\widehat{U}_E = \ldots \otimes \widehat{u}_E$ takes a nonentangled state of $\mathcal{S}^{(p)}$ to an entangled state, and $\widehat{u}_i^{(p)}$ takes $\mathcal{O}^{(p)}$ from a state of ignorance to a state of awareness β_i.

We take the state at the initial time t_0 to be one in which all four observers are ignorant and in which all three particles have spin up with respect to their respective z-axes:

$$|\psi_G, t_0\rangle = |\mathcal{O}^{(0)}; \gamma_0\rangle \left(\prod_{p=1}^{3} |\mathcal{O}^{(p)}; \beta_0\rangle\right) \left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_1\rangle\right). \tag{79}$$

Here we take the positive z axis for each particle to be in the direction of its motion, and the positive x axis perpendicular to the plane in which their motion lies (same direction for all particles).

The expected value of $\mathcal{O}^{(0)}$'s awareness at time t_3 is

$$\begin{aligned}\langle\psi_G, t_0|\widehat{G}(t_3)|\psi_G, t_0\rangle = \\ \gamma_1 \sum_{\substack{\{j,k,l\}\\ \text{odd } \#\ 1\text{'s}}} \left(\prod_{p=1}^{3} \langle\mathcal{S}^{(p)}; \alpha_1|\right) \widehat{u}_E^\dagger \left(\widehat{\Pi}^{(1)}_{j,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{k,\vec{n}^{(2)}} \otimes \widehat{\Pi}^{(3)}_{l,\vec{n}^{(3)}}\right) \widehat{u}_E \left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_1\rangle\right) + \\ \gamma_2 \sum_{\substack{\{j,k,l\}\\ \text{even } \#\ 1\text{'s}}} \left(\prod_{p=1}^{3} \langle\mathcal{S}^{(p)}; \alpha_1|\right) \widehat{u}_E^\dagger \left(\widehat{\Pi}^{(1)}_{j,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{k,\vec{n}^{(2)}} \otimes \widehat{\Pi}^{(3)}_{l,\vec{n}^{(3)}}\right) \widehat{u}_E \left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_1\rangle\right).\end{aligned} \tag{80}$$

If the eigenvalues γ_1 and γ_2 have the respective values 0 and 1, then the operator $\widehat{G}$ measures the probability of $\mathcal{O}^{(0)}$ determining that the $\mathcal{O}^{(p)}$'s observe an even number of spin-up particles during one run of the GHZM experiment:

$$\begin{aligned}P_{eu}(\vec{n}^{(1)}, \vec{n}^{(2)}, \vec{n}^{(2)}) = \\ \sum_{\substack{\{j,k,l\}\\ \text{even } \#\ 1\text{'s}}} \left(\prod_{p=1}^{3} \langle\mathcal{S}^{(p)}; \alpha_1|\right) \widehat{u}_E^\dagger \left(\widehat{\Pi}^{(1)}_{j,\vec{n}^{(1)}} \otimes \widehat{\Pi}^{(2)}_{k,\vec{n}^{(2)}} \otimes \widehat{\Pi}^{(3)}_{l,\vec{n}^{(3)}}\right) \widehat{u}_E \left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_1\rangle\right).\end{aligned} \tag{81}$$

4.2.1 Nonentangled Particles

We first consider the case in which an entangling interaction among the particles is absent, i.e.,

$$\widehat{u}_E = I_{\mathcal{S}}^{(1)} \otimes I_{\mathcal{S}}^{(2)} \otimes I_{\mathcal{S}}^{(3)}. \tag{82}$$

Then, using the above equation with (39), (40), and (81)

$$\begin{aligned}P_{eu}(\vec{n}^{(1)}, \vec{n}^{(2)}, \vec{n}^{(2)}) = \\ \cos^2(\theta^{(1)}/2)\cos^2(\theta^{(2)}/2)\sin^2(\theta^{(3)}/2) + \cos^2(\theta^{(1)}/2)\sin^2(\theta^{(2)}/2)\cos^2(\theta^{(3)}/2) + \\ \sin^2(\theta^{(1)}/2)\cos^2(\theta^{(2)}/2)\cos^2(\theta^{(3)}/2) + \sin^2(\theta^{(1)}/2)\sin^2(\theta^{(2)}/2)\sin^2(\theta^{(3)}/2),\end{aligned} \tag{83}$$

independent of the $\phi^{(p)}$'s. So, for *any* choices of the analyzer-magnet orientations $\vec{n}^{(p)}$ perpendicular to the particles' respective directions of motion ($\theta^{(p)} = \pi/2$),

$$P_{eu}(\phi^{(1)}, \phi^{(2)}, \phi^{(3)}) = 1/2. \tag{84}$$

4.2.2 Entangled Particles

On the other hand, if $\widehat{U}_E$ is such as to take the initial $\mathcal{S}^{(p)}$ state to the GHZM state, i.e.,

$$\widehat{u}_E \left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_1\rangle \right) = \left(\frac{1}{\sqrt{2}} \right) \left(\left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_1\rangle \right) - \left(\prod_{p=1}^{3} |\mathcal{S}^{(p)}; \alpha_2\rangle \right) \right), \tag{85}$$

the probability of $\mathcal{O}^{(0)}$ determining that an even number of spin-up measurements are made is, using the above equation with(39), (40), and (81),

$$P_{eu}(\vec{n}^{(1)}, \vec{n}^{(2)}, \vec{n}^{(2)}) = \left(1 + \cos(\phi^{(1)} + \phi^{(2)} + \phi^{(2)}) \sin(\theta^{(1)}) \sin(\theta^{(2)}) \sin(\theta^{(3)})\right) /2 \tag{86}$$

or, for $\theta^{(p)} = \pi/2$,

$$P_{eu}(\phi^{(1)}, \phi^{(2)}, \phi^{(3)}) = \left(1 + \cos(\phi^{(1)} + \phi^{(2)} + \phi^{(2)})\right) /2. \tag{87}$$

So an even number of spin-up measurements will never be found if one of the analyzers is oriented perpendicular to the plane of the particles' motion,

$$P_{eu}(0°, 90°, 90°) = P_{eu}(90°, 0°, 90°) = P_{eu}(90°, 90°, 0°) = 0, \tag{88}$$

but, contrary to the prediction (7) from instruction-set reasoning, an even number of spin-up measurements will *always* be found if all analyzers are oriented in the same sense perpendicular to this plane:

$$P_{eu}(0°, 0°, 0°) = 1. \tag{89}$$

5 Discussion

In the Heisenberg-picture formalism, the reason for the difference between the correlations of the observers' measurements in the nonentangled and entangled cases is the presence in the operators $\widehat{B}^{(p)}$, $\widehat{G}$ of different factors acting in subspaces of states pertaining, not to the observers but, rather, to the particles with which the observers have interacted by virtue of the measurements they've performed. These factors are in effect "labels" which become attached to the observers $\mathcal{O}^{(p)}$, $\mathcal{O}^{(0)}$ after they have undergone the local interactions $\widehat{U}^{(p)}_{M,\vec{n}^{(1)}}$, $\widehat{V}$. In the case that, prior to any measurements, the particles are subject to a local entangling interaction $\widehat{U}_E$, each of the particles $\mathcal{S}^{(p)}$ is labeled with a factor acting in the space of states of the other particle(s) with which it has interacted, so the label which becomes attached to $\mathcal{O}^{(p)}$ after measuring the corresponding particle $\mathcal{S}^{(p)}$ involves factors corresponding to the particle which the other observer(s) measured. In the end the observers compare their observations by means of another local interaction ($\mathcal{O}^{(1)}$

interrogates $\mathcal{O}^{(2)}$, $\mathcal{O}^{(0)}$ interrogates $\mathcal{O}^{(1)}$, $\mathcal{O}^{(2)}$, and $\mathcal{O}^{(3)}$), which has the effect of computing quantities such as (43) and (80).

The conceptual picture which emerges is thus the following: Interactions between entities label those entities. The labels consist of modifications to the Heisenberg-picture operators corresponding to the properties of the entities. Measurement-type interactions (20) transform the operators for the states of awareness of observers into sums of operators, each corresponding to a distinct state of awareness of the observer, and each labeled with factors corresponding to the system which the observer measured, as well as to other systems with which *that* system has previously interacted. These labels control the subsequent results of measurement involving the labeled operators, including in particular measurements of correlations between the states of awareness of observers who have measured particles which have previously interacted with one another.

Bell's theorem (5), (7) is avoided because the counterfactual reasoning which leads to it is not required and cannot be justified. In answer to the question "What is the mechanism which brings about these correlations?" there exists an answer other than the existence of instruction sets. Namely: When one of the observers performing, say, an EPRB experiment with both analyzer magnets oriented in the same direction measures the spin of one of the paired particles, that observer splits into noninteracting copies, each copy labeled with information corresponding to the states of the observed particle as well as to the state of the other particle. When the two observers—or, more precisely, the two pairs of observer-copies—exchange information about the results of their measurements, it is the attached labels which ensure that the "correct" copies of each of the observers interact; e.g., preventing two observer-copies who have both observed spin-up from communicating.

To be *completely* precise, we should say "two pairs of *sets* of observer-copies." This is necessary because, of course, the analyzer magnets need not be oriented in the same direction. When the analyzer magnets do not have the same orientation, there are four possible outcomes which the observers can experience, with (in general) unequal probabilities for the "same-spin" and "different-spin" cases. To avoid being led to the conclusion that our formalism erroneously implies equal probabilities for all outcomes regardless of the magnet orientations (Ballentine, 1973; Graham, 1973), we proceed along the lines of Deutsch's (1985) modification to the Everett interpretation and regard, for example, the third and fourth lines of eq. (44) as respectively representing continuous infinities of identical observers $\mathcal{O}^{(1)}$ and $\mathcal{O}^{(2)}$. The two terms in eq. (64) or eq. (65) then represent continuous infinities of two different types of observers ("saw-up" and "saw-down"), and the four terms in the operator in the second member of eq. (66) represent continuous infinities of four different types of pairs of observers ("saw-up/saw-up," "saw-up/saw-down," etc.). The relative number of each type, as well as the specific nature of each type (states of awareness of the observers), is governed by the expectation value of the corresponding term in the initial state $|\psi_0, t_0\rangle$.

So, the splitting of each observer into copies at each measurement interaction is represented by the local dynamics of the operators describing their states of awareness relative to what they were at the initial time t_0; in particular, the possibilities for interaction of observers of entangled systems are determined by the labels attached to the operators. Determination of the number of each type of observer-copy produced at each splitting, as well as the specific state of awareness of each type of observer-copy, involves information

about the initial conditions of the system, information which in the Heisenberg picture is contained in the time t_0 state vector. (DeWitt (1998) emphasizes that quantum systems are "described jointly by the dynamical variables and the state-vector.") Just as observers or other entities may be regarded as receiving and carrying with them, in a local manner, the labels described above, they may also be envisioned as carrying with them in a similarly local manner the requisite initial-condition information.

Since one cannot argue for the existence of counterfactual instruction sets, the conditions of Bell's theorem do not apply. Had angles other than those that actually were used been chosen for the analyzer magnets, copies of each observer carrying labels appropriate to those angles would have resulted. There are indeed "instruction sets" present; but they determine, not the results of experiments which were not performed but, rather, the possibilities for interaction and information exchange between the Everett copies of the observers who have performed the experiments.

Bohr's reply to EPR can also be reinterpreted in the present context. Regarding correlations at a distance, Bohr (1935) states that "of course there is in a case like that just considered no question of a mechanical disturbance of the system under investigation during the last critical stage of the measuring procedure. But even at this stage there is essentially the question of *an influence on the very conditions which define the possible types of predictions regarding the future behavior of the system."* The Everett splitting and labeling of each observer constitutes just such an influence, determining the possible types of interactions with physical systems and observers which the observer can experience in the future without in any way producing a "mechanical disturbance" of distant entities.

The Everett interpretation in the Heisenberg picture thus removes nonlocality from the list of conceptual problems of quantum mechanics. The idea of viewing the tensor-product factors in the Heisenberg-picture operators as in some sense "literally real" introduces, however, a conceptual problem of its own.[3] Entanglement via the introduction of nontrivial "label" factors is not limited to interactions between two or three particles; each particle of matter is labeled, for eternity, by all the particles with which it has ever interacted. *What is the physical mechanism by means of which all of this information is stored?*

The issue of "where the labels are stored" may seem less problematic in the context of the Everett interpretation of Heisenberg-picture quantum field theory. After all, in quantum field theory, operators corresponding to each species of particle and evolving according to local differential equations already reside at each point in spacetime. (In the EPRB and GHZM experiments the particles in question are considered to be distinguishable and so may be treated, for purposes of analyzing the experiments, as quanta of different fields. More complicated objects, such as observers and magnets, might be approximated as excitations of effective composite fields, following, e.g., Zhou et al. (2000).)

Even in the event that such a program for a literal, indeed mechanistic picture of measurement in quantum field theory cannot be realized, it remains the case that Everett's model for measurement in the Heisenberg picture provides a quantum formalism which is explicitly local and in which the problem of Bell's theorem does not arise.

[3]The fact that the precise details of representation of the Heisenberg-picture operators depend, e.g., on the choice of initial time t_0 (d' Espagnat, 1995, Section 10.8) should *not* be a problem in viewing them as "real," any more than, e.g., the fact that the components of the electromagnetic stress tensor depend on the choice of Lorentz frame.

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